Every Student Counts

Middle School
Professional Development Guide
Year 2 - Day 2

Iowa Department of Education

Middle School Session –Facilitator Plan
Year 2 - Day 2

Content Goal:

NCTM Geometry Standard
Use visualization, spatial reasoning, and geometric modeling to solve problems
NCTM Measurement Standard
Apply appropriate techniques, tools, and formulas to determine measurements

Principle Focus: Technology

Process Focus: Communication
Representation
Problem Solving

Overall Teaching Goal: Teaching and learning mathematics through problem solving

Activity	Description for Facilitator	Time (Min)	Teacher Masters (TM) & Materials
1. Welcome and opening	1. Welcome 2. Review Year 2 Outline 3. Review Daily Overview for Day 2 4. Review Day 2 agenda 5. React to technology • PSSM (pp. 24 – 27) • How can the use of technology impact how mathematics is taught? • How can the use of technology impact what mathematics is taught? 6. View State ESC website • www.state.ia.us/educate/ecese/is/esc/index.hdml • user: ESCounts • Password: PBITSMDP	50	**TM 1:** Year Two Outline **TM 2:** Daily Overview **TM 3:** Year 2 Day 2 Agenda **TM 4:** Reflecting on Technology • Principles and Standards for School Mathematics (PSSM) • PSSM Quick Reference Guide
2. Meaningful Distributed Practice	1. Meaningful Distributed Practice – Perimeter/Area 2. Do first MDP activity	30	**TM 5:** MDP Activities (to be handed out after the activity)

Activity	Description for Facilitator	Time (Min)	Teacher Masters (TM) & Materials
	3. Participants develop possible sequence of MDP activities with explanation of why next activity is used 4. Show rest of MDP developed for this class		**TM 6:** MDP Overhead Activity Sheets **TM 7:** MDP Components
3. Problem-Based Instructional Task – Perimeter/Area	1. Pet Fence problem 2. Discuss extension PBIT from lesson plan 3. Formative Assessment: Evaluating Student Work • Review Black Box article • Goal – provide feedback to student • Develop rubric on chart paper (0 to 6 point scale) • Score student work (Students from Larry's and Cynthia's classes) • Label student work • Discuss scoring • Discuss key learnings from area and perimeter problem	110	**TM 8:** Lesson Plan: Perimeter/Area (maximum area for given length) **TM 9:** Lesson Plan: Perimeter/Area (maximum area for given perimeter) **TM 10:** Dog Pen Problem (42 feet) **TM 11:** Dog Pen Problem (40 feet) **TM 12:** Pet Fence Problem (42 feet) **TM 13:** Student Work **TM 14:** Scoring Rubric • Graphing calculator • Graph paper • Student papers • Rubric
4. Debrief van Hiele reading	1. M Y2 D2 PowerPoint can be used for activities 4 and 5 2. At each level of the van Hiele model, describe what the student can do and something they can not do. 3. At each level of the van Hiele model, identify one or two types of activities and questions that will help students develop to another level.	15	**TM 15:** Reflecting on van Hiele

5. Spatial Visualization Activities	1. M Y2 D2 PowerPoint can be used for activities 4 and 5 2. Review and discuss NCTM Principles and Standards – Reread Visualization etc. (pp. 237-239) 3. Build with cubes on a building mat 4. Look at base outline and then build from base plan 5. Draw a building plan (base outline as well as front, back, right, left, using reflector) 6. Build from building plan 7. Draw a cube using isometric paper 8. Draw all possible 3 cube buildings using isometric paper (extension – what are all the 4 cube buildings?) 9. Find maximum and minimum number of cubes that can be used to build a structure 10. Show the NCTM *Illuminations* website where you can examine the drawings 11. Introduce to *Ruins of Montarek* website by Connections	105	**TM 16:** Building Mat **TM 17:** Base Outline **TM 18:** Base Plan **TM 19:** Building and Drawing Building Plans **TM 20:** Building Plan 1 **TM 21:** Building Construction **TM 22:** Isometric Explorations (pp. 113 – 114 in Navigations book and on Navigations CD) **TM 23:** Do They Match (from NCTM Illuminations) **TM 24:** Reflecting on the Video • Cubes • Georeflector • Isometric graph paper • Computer with internet hookup • Isometric and grid graph paper available at two web sites (http://www.mathematicshelpcentral.com/ or http://www.mathsphere.co.uk/Resources/MathSphereFreeGraphPaper.shtml • Link to http://illuminations.nctm.org/index.aspx • Link to Illuminations Lesson http://illuminations.nctm.org/index_o.aspx?id=166

			▪ Link to Video Library http://www.learner.org/resources/series33.html Link to Student Work http://www.learner.org/channel/courses/teachingmath/grades6_8/ ▪ Link to Ruins of Montarek http://www.math.msu.edu/mathed/mth202F04/ruins/
6. Closure	• Review day's agenda • Review assignment for January 31/February 1	10	**TM 1**: Overview **TM 25**: Assignment **TM 26**: Evaluation

Facilitator's Tool for Planning the Session

What is the background reading?

1. Read intro in navigation book on visualization, spatial reasoning and modeling (pp. 59-60)

2. *NCTM Principles and Standards* – Technology (pp. 24-27)

3. *NCTM Principles and Standards* – Reread Visualization etc. (pp. 237-239)

4. Crowley, Mary L. "The van Hiele Model of the Development of Geometric Thought." in CD that comes with *Navigating through Geometry in Grades 6 – 8*

What equipment and materials should **participants** bring?

- Computer
- Cubes
- Georeflector - Red
- Graphing Calculator
- Navigations book and CD
- Principles and Standards for School Mathematics (PSSM)
- Ruler

What Teaching Masters need to be copied?

Handouts:

TM 1: Year Two Outline
TM 3: Year 2 Day 2 Agenda
TM 4: Reflecting on Technology

TM 5: MDP Activities (to be handed out after the activity)
TM 7: MDP Components
TM 8: Lesson Plan: Perimeter/Area (maximum area for given length)
TM 9: Lesson Plan: Perimeter/Area (maximum area for given perimeter)
TM 10: Dog Pen Problem (42 feet)
TM 11: Dog Pen Problem (40 feet)
TM 12: Pet Fence Problem (42 feet)
TM 13: Student Work
TM 14: Scoring Rubric
TM 15: Reflecting on van Hiele
TM 16: Building Mat
TM 17: Base Outline
TM 18: Base Plan
TM 19: Building and Drawing Building Plans
TM 20: Building Plan 1
TM 21: Building Construction
TM 22: Isometric Explorations (pp. 113 – 114 in Navigations book and on Navigations CD)
TM 23: Do They Match (from NCTM Illuminations)
TM 24: Reflecting on the Video
TM 25: Assignment
TM 26: Evaluation

What Teaching Masters need to be copied for presenters?

TM 1: Year Two Outline
TM 2: Daily Overview
TM 3: Year 2 Day 2 Agenda
TM 4: Reflecting on Technology
TM 5: MDP Activities
TM 6: MDP Overhead Activity Sheets
TM 7: MDP Components
TM 8: Lesson Plan: Perimeter/Area (maximum area for given length)
TM 9: Lesson Plan: Perimeter/Area (maximum area for given perimeter)
TM 10: Dog Pen Problem (42 feet)
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TM 16: Building Mat
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TM 22: Isometric Explorations (pp. 113 – 114 in Navigations book and on Navigations CD)
TM 23: Do They Match (from NCTM Illuminations)
TM 24: Reflecting on the Video
TM 25: Assignment
TM 26: Evaluation

Teaching supplies/materials/technologies

- Computer with internet hookup
- Cubes
- Georeflector
- Graph paper
- Graphing calculator
- Principles and Standards for School Mathematics (PSSM)
- PSSM Quick Reference Guide
- Rubric
- Student papers
- Isometric and grid graph paper available at two web site
  - http://www.mathematicshelpcentral.com
  - http://www.mathsphere.co.uk/Resources/MathSphereFreeGraphPaper.shtml
- Link to State website
  - www.state.ia.us/educate/ecese/is/esc/index.hdml
- Link to Illuminations
  - http://illuminations.nctm.org/index.aspx
- Link to Illuminations Lesson
  - http://illuminations.nctm.org/index_o.aspx?id=166
- Link to Video Library
  - http://www.learner.org/resources/series33.html
- Link to Student Work
  - http://www.learner.org/channel/courses/teachingmath/grades6_8/
- Link to Ruins of Montarek
  - http://www.math.msu.edu/mathed/mth202F04/ruins/

Activity 1: Welcome and Opening

Time: 50 minutes

Overview and Rationale:
This activity connects the day with the goals for the year. It will provide an opportunity to relate daily activities to the year-long goals and activities.

Conducting the Activity:

1. Year 2 Outline Chart TM 1
   - Remind participants of the big picture for the year
   - Point out where we've been and where we're going
   - Emphasize the NCTM Content Standards, Principles and Process Standards of the day

2. Go through Year 2 Day 2 Agenda TM 3 handout while using the Day 2 Overview Chart TM 2
   - Briefly go through agenda
   - Remind participants of the main themes of Every Student Counts
   - Point out how those themes will be applied to the goals and focus areas
   - Use the Quick Reference Guide to locate the NCTM Standards being highlighted

3. Discuss Principle Focus
   - Participants fill in TM 4 – Reflecting on Technology
   - Discuss at table
   - Share a few ideas from each team with the whole group

4. View State Website

Materials

TM 1: Year Two Outline
TM 2: Daily Overview
TM 3: Year 2 Day 2 Agenda
TM 4: Reflecting on Technology

- Principles and Standards for School Mathematics (PSSM)
- PSSM Quick Reference Guide

TM 1

Year 2 Outline
2005-2006

	Day 1 October 4/5	Day 2 November 8/9	Day 3 January 31/February 1	Day 4 April 11/12
NCTM Content Standard	Geometry	Geometry	Geometry	Geometry
	Analyze characteristics and properties of 2 and 3-dimensional shapes and develop mathematical arguments about geometric relationships	Use visualization, spatial reasoning, and geometric modeling to solve problems	Specify locations and describe spatial relationships using coordinate geometry and other representational systems	Apply transformations and use symmetry to analyze mathematical situations
NCTM Content Standard 2	Geometry	Measurement	Measurement	
	Use visualization, spatial reasoning, and geometric modeling to solve problems	Apply appropriate techniques, tools, and formulas to determine measurements	Apply appropriate techniques, tools, and formulas to determine measurements	
NCTM Content Standard 3	Measurement			
	Understand measurable attributes of objects and the units, systems, and processes of measurement			
Mathematical Activities	Analyzing characteristics and properties of polygons	Use visualization, spatial reasoning, and geometric modeling Maximizing and minimizing area, perimeter and volume	Use coordinate geometry to represent and examine the properties of geometric shape	Apply transformations and use symmetry to analyze mathematical situation
NCTM Principle	Equity	Technology	Teaching	Learning

	Day 1 **October 4/5**	**Day 2** **November 8/9**	**Day 3** **January 31/February 1**	**Day 4** **April 11/12**
NCTM Process Standard	Communication Reasoning and Proof Problem Solving	Communication Representation Problem Solving	Connections Problem Solving	Problem Solving Connections
Assessment	Teacher Observation Checklist to provide Feedback to the Students Questioning	Rubric Collecting Samples of Participants' Work Analyzing Student Work to Provide Feedback to the Students	Formative use of summative assessment Examining ITBS and classroom summative assessment	Peer Assessment
Technology/ Manipulative Tools	Sketchpad Navigation CD Computer (participants bring) Geostrips	Graphing Calculator Navigation CD – Applet Cubes Web Page: http://illuminations.nctm.org/index.aspx.	Graphing Calculator Geoboards Geometry Sketchpad	Applet Sketchpad Geoboard Georeflector Computer (participants bring)

TM 2

Teach for Understanding and Focus on Meaning

Problem-Based Instructional Tasks Teaching through Problem Solving	Meaningful Distributed Practice of Concepts, Skills, & Problem Solving

Today's Goals . . .

Content Goal: Geometry and Measurement
Principle Goal: Technology
Process Goal: Communication, Reasoning and Proof, Problem Solving

Today's Objectives . . .

- *Use visualization, spatial reasoning, and geometric modeling to solve problems about geometric relationships*
- *Understand measurable attributes of objects and the units, systems, and processes of measurement*

TM 3

Year 2 Day 2 Agenda

Content Goals:

NCTM Geometry Standards

- Use visualization, spatial reasoning, and geometric modeling to solve problems about geometric relationships

NCTM Measurement Standard

- Understand measurable attributes of objects and the units, systems, and processes of measurement

Principle Focus: Technology

Process Focus: Communication
Reasoning and Proof
Problem Solving

Agenda

- Welcome and opening
- Meaningful Distributed Practice
- Problem-Based Instructional Task (Perimeter/Area)
- Review Student Work
- Debrief Readings
- Spatial Visualization
- Closure

Assignments for January 31/February 1:

1. *NCTM Principles and Standards* – Teaching (pp. 16-19)
   - How does Every Student Counts promote the Teaching Principle?

2. *NCTM Principles and Standards* – Geometry (pp. 232-239)
   - What coordinate geometric concepts do middle school students generally understand when they leave elementary school?
   - What types of activities will help extend the middle school students' understanding of coordinate geometry?

3. Slavit, David. "Above and beyond AAA: The Similarity and Congruence of Polygons." on CD from *Navigating through Geometry in Grades 6 – 8* – Article is under section called "More Readings."
   - What impact does this article suggest for curriculum being taught?
   - What impact does this article suggest for instruction?

   - Do one of the following activities:
     - Teach a middle school geometry PBIT – either one done in the workshops or one out of the *Navigating through Geometry in Grades 6 – 8* book. Bring samples of student work from the lesson showing high, middle, and low levels of achievement.
     - Interview 3 or 4 middle school students about geometric understandings – Bring a listing of student comments and/or work activities that illustrate students working at the different van Hiele levels.

Resources to bring on January 31/February 1:

1. Computer with Geometer's Sketchpad installed

2. Plastic Geoboard

3. Navigating through Geometry in Grades 6 – 8 book and CD

4. Principles and Standards for School Mathematics (PSSM)

TM 4

Reflecting on Technology

(pp. 24 – 27 PSSM)

- How can the use of technology impact how mathematics is taught?

- How can the use of technology impact what mathematics is taught?

Possible Responses Reflecting on Technology

(pp. 24 – 27 PSSM)

How can the use of technology impact how mathematics is taught?

- Quick calculations
- More thinking
- Quickly organize data
- Level playing field
- See changes in graphs more quickly and in better form
- More differentiation
- Access to visual models
- Teacher choice on whether or not to use technology
- Allows students to work with larger numbers

How can the use of technology impact what mathematics is taught?

- Help look at linear relationships as a pre-algebra activity
- More time on thinking
- Focus more deeply on problems
- More level 2 activities
- More representations
- Reflection and Reasoning
- Connect ideas
- Think through reasoning
- Visualization possible

Activity 2: Meaningful Distributed Practice (MDP)

Time: 30 minutes

Overview and Rationale

This activity provides practice with Meaningful Distributed Practice. It connects MDP with geometry goals and assessment.

Conducting the Activity

1. Begin with the first distributed practice example. Present it as a teacher presenting to a class. Have participants share 2 (or so) examples of work. After sharing out, step out of teacher mode and, as a presenter, ask the participants what they thought purpose was for asking this distributed practice. Why were the specific questions asked and why did particular participants share?
2. Share the second example of distributed practice, but do not present it as a teacher would. However, ask participants the purpose of the question and what important ideas to pull out of student discussion.
3. Have participants do the third distributed practice and look at the fourth distributed practice. Process both in a similar fashion to the first two problems.
   - Save the last two distributed practice problems for after the PBIT. Ask the participants to view the distributed practice in terms of formative assessment information. The first several would determine if students had sufficient background. The fifth problem would assess if the students met the goals of the PBIT and the last problem would check to see if students could extend the process and use it in a new way.
4. Review the components of MDP. Ask if the activities used here fit the components of MDP.
5. Take out geometry standards in back of PSSM. Look at visualization.
   - What do we teach in middle school mathematics that applies to those areas? (e. g. surface area; rectangular prisms; volume; prisms; views; 2D and 3D; nets)
   - Work in table teams to develop a sequence of activities around volume standard.
   - Share

Materials

TM 5: MDP Activities (to be handed out after the activity)
TM 6: MDP Overhead Sheets
TM 7: MDP Components

TM 5

Distributed Practice, Questions, and Assessment: Grade Level/Class ___Middle School
Big Idea(s) _____Perimeter and Area

Day One	Day Two	Day Three
Practice Activity 1 Draw a rectangle with width 4 and length 7. Find the perimeter and area of the rectangle. Draw a model that represents the perimeter and the area.	Practice Activity 2 Draw a rectangle with a width of 6 and a length of twice that. Mentally find the perimeter and area of the rectangle.	Practice Activity 3 Draw a rectangle with a perimeter of 24. What's the area of your rectangle?
Questions: What is the perimeter? How did you determine this? What is the area? What models did you use for perimeter? for area? What labels are appropriate for perimeter and area? Why units and square units?	Questions: What is the perimeter? How did you determine that? What is the area? How can you mentally determine the perimeter and area?	Questions: Have each student state the area of his/her rectangle and have other students guess the dimensions. How did you determine your dimensions?

TM 5 (continued)

Day Four	Day Five	Day Six
Practice Activity 4 Draw a rectangle with a perimeter of 28 units. What's the area of your rectangle?	Practice Activity 5 Draw a rectangle with a perimeter of 36 that has the largest possible area.	Practice Activity 6 Draw a rectangle with an area of 36 with the largest possible perimeter.
Questions: How did you decide on your dimensions? What student's rectangle has the largest area?	Questions: Share dimensions of various rectangles and determine largest area.	Questions: Determine the rectangle with the largest perimeter.

TM 6

Meaningful Distributed Practice Activities

Day 1: Draw a rectangle with width 4 units and length 7 units. Find the perimeter and area of the rectangle. Draw a model that represents the perimeter and the area.

Day 2: Draw a rectangle with a width of 6 and a length twice that. Mentally find the perimeter and area of the rectangle.

TM 6 (Continued)

Meaningful Distributed Practice Activities

Day 3: Draw a rectangle with a perimeter of 24 units. What's the area of your rectangle?

Day 4: Draw a rectangle with a perimeter of 28 units. What's the area of your rectangle?

TM 6 (Continued)

Meaningful Distributed Practice Activities

Day 5: Draw a rectangle with a perimeter of 36 that has the largest possible area.

Day 6: Draw a rectangle with an area of 36 with the largest possible perimeter.

TM 7

Meaningful Distributed Practice of Concepts, Skills and Problem-Solving

- Targets an identified need based on multiple data sources
- Helps students develop a deep understanding of a *Big Idea*
- Helps students develop flexibility and fluency with skills and concepts
- Builds on and extends understanding
- Uses problems and activities that help students learn to use multiple representations and learn to use multiple reasoning strategies
- Uses problems from a variety of contexts so students learn to make connections.

Revised September 2005

Activity 3: Problem-Based Instructional Task: Perimeter/Area

Time: 110 minutes

Overview and Rationale

This activity will use geometry and measurement content to show a PBIT as well as showing formative assessment by analyzing student work.

Conducting the Activity

1. Fence Problem
   - Complete Pet Fence problem with 40 or 42 feet of fence.
   - Make a table or graph containing information needed to solve the problem
   - Use graphing calculator to solve the problem
   - Different groups share solutions
2. Formative Assessment: Evaluating Student Work
   - Review Black Box article (Feedback through Grading)
     - Written tasks (along with oral questioning) should encourage students to develop and show understanding of key features of what they've learned
     - Comments should identify what has been done well and what still needs improvement and give guidance on how to make that improvement
     - Opportunities for students to respond to comments should be planned as part of the overall learning process
   - Examine student work (students from special and regular education classes)
     - Determine feedback that could be given to each of these students
3. Examine rubric in relationship to the student papers

Materials

TM 8: Lesson Plan: Perimeter/Area (maximum area for given length)
TM 9: Lesson Plan: Perimeter/Area (maximum area for given perimeter)
TM 10: Dog Pen Problem (42 feet)
TM 11: Dog Pen Problem (40 feet)
TM 12: Pet Fence Problem (42 feet)
TM 13: Student Work
TM 14: Scoring Rubric

- Graph paper available at two web sites (http://www.mathematicshelpcentral.com/ or http://www.mathsphere.co.uk/Resources/MathSphereFreeGraphPaper.shtml
- Graphing Calculator

TM 8

PROBLEM-BASED INSTRUCTIONAL TASK
LESSON PLAN I

OBJECTIVE/BENCHMARK:

Geometry

- Analyze characteristics and properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships
  - Understand relationships among side lengths, perimeters and areas

Measurement

- Understand measurable attributes of objects and the units, systems and processes of measurement
  - Understand, select, and use units of appropriate size and type to measure angles, perimeter, and area

TITLE: Maximizing Area for a Given Length – Pet House

GRADE LEVEL/COURSE: Middle School

PRE-REQUISITE KNOWLEDGE:

Knowledge of perimeter
Knowledge of area

NCTM STANDARD(S): (Shaded)

NCTM Content Standards →	Number & Operations	Algebra	Geometry	Measurement	Data Analysis & Probability
NCTM Process Standards →	Problem Solving	Reasoning & Proof	Communication	Connections	Representation

MATERIALS NEEDED:

Audio-visual: Overhead of the problem

Manipulatives/Materials:

- Teaching Master (TM) Dog Pen Problem

Literature:
Technology/Software: TI73 calculators
Other:

MAIN LESSON DEVELOPMENT:

Launch:
Introduce the problem and show the students the overhead of the problem and table. Explain that the length is always 42 feet. Have a student come forward and draw in the dimensions of one possible rectangular area (i.e. 8ft by 26ft). Have the student say what he/she knows about the amount of fence used. (It is equal to 42 feet.) Ask what is the area of that pen? (8 x 26 = 208 sq. ft.).

Explore:
Organize the participants in groups of 2 or 3. Hand each a problem sheet. They are to make a table containing the information they create to solve the problem. Observe the groups as they attempt to find the largest possible area. Are they using some systematic way (e.g. changing the width by one unit each time) to ensure they have found all possible rectangles? In each case, is the fence length equal to 42 feet? Are they creating new rectangles based on their previous choices (selecting dimensions close to those of the rectangles with the largest areas)?

Summarize:
Aside: Since the rectangle with the largest area involves dimensions that are not whole numbers (10.5 x 21), the teacher should state, upon seeing rectangles with apparent large areas, "There is a rectangle with a larger area." Do not tell the students to try a fraction, but rather let them discover that by themselves.
Have different groups share their findings. Have several groups of students share the process they used to determine the rectangle with the largest volume. Ask students if the fence length is 42 feet. Ask students if the area changed and was that what they expected to happen. When is the area the smallest? Ask students how they can be sure that the rectangle they say has the largest area is the correct answer. If no group determined the correct answer, state that there is another rectangle that has a larger area. Look at the dimensions of the rectangle that the class thinks gives the largest area. Ask the class to offer suggestions for other dimensions. Try them and see if the area is indeed larger.

MODIFICATIONS/EXTENSIONS:

Questions to ask as participants are working.

For struggling students:

- If the dimensions of the pen where 5 x 16, what would the fence length be? What would the area be? There is a rectangle with a larger area. What should you try next?

- Do you think there is a rectangle with a larger area than those you have listed? What dimensions might you try next?

- Change the length of the fence to 40 feet. Then the numbers are easier to deal with.

For advanced students:

- Draw a graph relating the lengths to the various areas.
- Change the length of the fence to be 41 feet. Now it is a greater challenge (the largest area occurs when the rectangle is 10.25 x 20.5.)
- See if the students can determine a general formula for determining the largest area given a fence of any length (a square with a side 1/4 the length for the perimeter).
- The largest area possible for the pen would actually be a circle. What would that area be?

Use of technology

Introduce the TI73 to the students. In particular focus on the LIST button and the GRAPH function.

- Push the LIST button and input the length of each rectangle into L1 and the corresponding area into L2.
  - Make sure that for each length there is one area and vice versa.
  - After all the lengths and area have been inputted, Push the Y= button and clear any equations that might be there.
  - Push 2nd Y= and turn off all the plots (#4).
- Turn on PLOT 1.
- Select the scatter plot graph (the first one).
  - The X list should be L1 and the Y list should be L2.
  - The mark should be the square.
  - The WINDOW button should be pushed and the proper settings denoted.
  - Then the data contained in L1 and L2 (length and area) can be graphed.
  - Use trace button to access specific points on the calculator

You could have the students graph the data on graph paper and compare the hand made graph to that on the calculator. Specific points on the calculator graph can accessed by pushing the TRACE button then the left and right arrows. Specific comparisons between the two graphs can be made point by point.

Ask questions about the shapes of the graphs. Do the shapes indicate that the maximum area has been correctly identified? How do you know?

CHECKING FOR UNDERSTANDING (FORMATIVE ASSESSMENT)

What will you assess?

- Knowledge of the perimeter of a rectangle
- Knowledge of area of a rectangle
- Ability to look for patterns and select appropriate new choices based on the previous results

How will you assess it?

- During student discussion time, teacher observation will determine whether students can correctly determine perimeters and areas of rectangles
- Observation of group work and the involvement of all members

---------- (REFLECTION AFTER TEACHING THE LESSON) ----------

- **How did the students perform?**
- **How will you use this information to guide future instructional decisions?**

TM 9

PROBLEM-BASED INSTRUCTIONAL TASK
LESSON PLAN II

OBJECTIVE/BENCHMARK:

Geometry

- Analyze characteristics and properties of two-dimensional geometric shapes and develop mathematical arguments about geometric relationships
  - Understand relationships among side lengths, perimeters and areas

Measurement

- Understand measurable attributes of objects and the units, systems and processes of measurement
  - Understand, select, and use units of appropriate size and type to measure angles, perimeter, and area

TITLE: Maximizing Area for a Given Perimeter

GRADE LEVEL/COURSE: Middle School

PRE-REQUISITE KNOWLEDGE:

Knowledge of perimeter
Knowledge of area

NCTM STANDARD(S): (Shaded)

NCTM Content Standards →	Number & Operations	Algebra	Geometry	Measurement	Data Analysis & Probability
NCTM Process Standards →	Problem Solving	Reasoning & Proof	Communication	Connections	Representation

MATERIALS NEEDED:

Audio-visual: Overhead of the problem

Manipulatives/Materials:

- Teaching Master (TM) Dog Pen Problem

Literature:
Technology/Software: TI73 calculators
Other:

MAIN LESSON DEVELOPMENT:

Launch:
Introduce the problem and show the students the overhead of the problem and table. Explain that the perimeter is always 42 feet. Have a student come forward and draw in the dimensions of one possible rectangular area (i.e. 8ft by 13ft). Have the student say what he/she knows about the amount of fence used. (It is equal to 42 feet.) Ask what is the area of that pen? (8 x 13 = 104 sq. ft.).

Explore:
Organize the participants in groups of 2 or 3. Hand each a problem sheet. They are to make a table containing the information they create to solve the problem. Observe the groups as they attempt to find the largest possible area. Are they using some systematic way (e.g. changing the width by one unit each time) to ensure they have found all possible rectangles? In each case, is the perimeter equal to 42 feet? Are they creating new rectangles based on their previous choices (selecting dimensions close to those of the rectangles with the largest areas)?
Aside: Since the rectangle with the largest area involves dimensions that are not whole numbers (10.5 x 10.5), the teacher should state, upon seeing rectangles with apparent large areas, "There is a rectangle with a larger area." Do not tell the students to try a fraction, but rather let them discover that by themselves.

Summarize:
Have different groups share their findings. Have several groups of students share the process they used to determine the rectangle with the largest area. Ask students if the perimeter is 42 feet. Ask students if the area changed and was that what they expected to happen. When is the area the smallest? Ask students how they can be sure that the rectangle they say has the largest area is the correct answer. If no group determined the correct answer, state that there is another rectangle that has a larger area. Look at the dimensions of the rectangle that the class thinks gives the largest area. Ask the class to offer suggestions for other dimensions. Try them and see if the area is indeed larger.

MODIFICATIONS/EXTENSIONS:

Questions to ask as participants are working.

For struggling students:

- If the dimensions of the pen where 5 x 16, what would the perimeter be? What would the area be? There is a rectangle with a larger area. What should you try next?

- Do you think there is a rectangle with a larger area than those you have listed? What dimensions might we try next?

- Change the perimeter of the fence to 40 feet. Then the numbers are easier to deal with.

For advanced students:

- Draw a graph relating the lengths to the various areas.
- Change the perimeter to 41 feet. Now it is a greater challenge (the largest area occurs when the rectangle is 10.25 x 10.25.
- See if the students can determine a general formula for determining the largest area given a fence of any length (a square with a side 1/4 the length for the perimeter).
- The largest area possible for the pen would actually be a circle. What would that area be?

Use of technology

Introduce the TI73 to the students. In particular focus on the LIST button and the GRAPH function.

- Push the LIST button and input the length of each rectangle into L1 and the corresponding area into L2.
  - Make sure that for each length there is one area and vice versa)
  - After all the lengths and area have been inputted, Push the Y= button and clear any equations that might be there.
  - Push 2nd Y= and turn off all the plots (#4).
- Turn on PLOT 1.
- Select the scatter plot graph (the first one).
  - The X list should be L1 and the Y list should be L2.
  - The mark should be the square.
  - The WINDOW button should be pushed and the proper settings denoted.
  - Then the data contained in L1 and L2 (length and area) can be graphed.
  - Use trace button to access specific points on the calculator

You could have the students graph the data on graph paper and compare the hand made graph to that on the calculator. Specific points on the calculator graph can accessed by pushing the TRACE button then the left and right arrows. Specific comparisons between the two graphs can be made point by point.

Ask questions about the shapes of the graphs. Do the shapes indicate that the maximum area has been correctly identified? How do you know?

CHECKING FOR UNDERSTANDING (FORMATIVE ASSESSMENT)

What will you assess?

- Knowledge of the perimeter of a rectangle
- Knowledge of area of a rectangle
- Ability to look for patterns and select appropriate new choices based on the previous results

How will you assess it?

- During student discussion time, teacher observation will determine whether students can correctly determine perimeters and areas of rectangles
- Observation of group work and the involvement of all members

---------- **(REFLECTION AFTER TEACHING THE LESSON)** ----------

- **How did the students perform?**
- **How will you use this information to guide future instructional decisions?**

TM 10

Dog Pen Problem Name _____

A dog trainer wants to make the largest possible pen for his dogs. He has 42 feet of fence. What is the largest area the pen can have?

Length	Width	Perimeter	Area

The rectangle with the largest area has a length of _____ and a width of _____.

TM 11

Dog Pen Problem Name _____

A dog trainer wants to make the largest possible pen for his dogs. He has 40 feet of fence. What is the largest area the pen can have?

Length	Width	Perimeter	Area

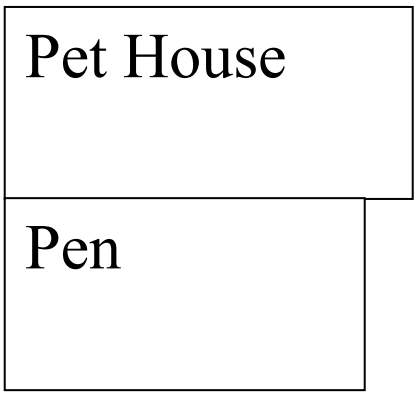
The rectangle with the largest area has a length of _____ and a width of _____.

TM 12

Pet Fence Problem

Name _____

You would like to make an enclosed rectangular pen for your pet. You do not need any fencing along the wall of the pet house. You have 42 feet of fencing and the pet house is 28 feet long. Make a table that indicates the area of the pen for various lengths and widths.



The largest area occurs when the length is _____ and the width is _____.